

# Communication and Optimization for Satellite Attitudes Using Proportional-Integral-Derivative Controller

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**Abstract**— In the proposed research work to determination of attitude and optimization of control parameters for attitude control and stabilization system. The equations as given to transform Euler angles to Quaternions and vice versa. PID Controller comprises is control loop feedback mechanism. The modelling (MATLAB-SIMULINK) for control parameters are defined and optimized to be actuated via reaction wheels in conjugation with Magneto Torquer. The attitude of the model is determined and then the control equations are adapted so that attitude as stabilized by reaction wheel and the torque rod achieves momentum damping. The stability of the system is demonstrated using Lyapunov function. Numerical simulations has performed on MATLAB to optimize control parameters. Thus, the feasibility and effectiveness of the PID control system as demonstrated.

**Keywords**— Quaternion, PID, Wahba's Loss Function, Lyapunov function

## I. INTRODUCTION

The Quaternion method to obtain the attitude matrix, which as used to determine the current attitude of the system. It, briefly describes the optimization of Attitude determination and control system (ADCS) for Low Earth Orbit satellites. This work is about the optimization techniques of ADCS components of a satellite using PID controller [1], [2].

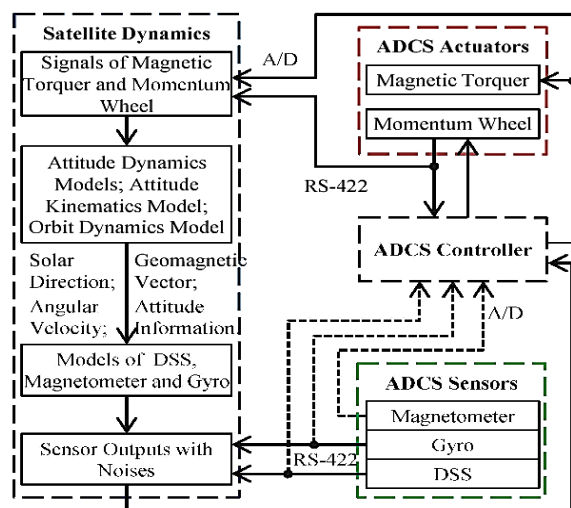


Figure. 1. ADCS Block Diagram [3]

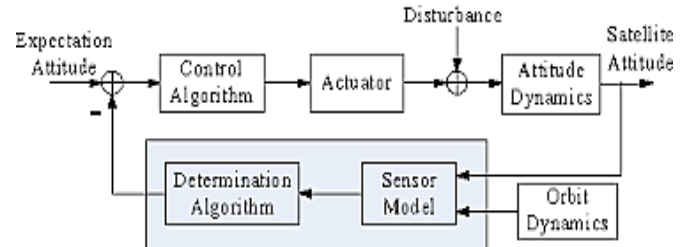


Figure. 2. Attitude control system

Euler's angles for simplification: (See Equation (1) & (2))

$$\text{Rotation Matrix (RM)} = \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\phi & 0 & -s\phi \\ 0 & 1 & 0 \\ s\phi & 0 & c\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & s\theta \\ 0 & -s\theta & c\theta \end{bmatrix} \quad (1)$$

$$\frac{d(RM)}{dt} = RM \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad (2)$$

C=Cosine terms & S Sine terms

Thus, by equating the parametric form of RM with the above relation, we can obtain the relation between angular velocity and rotation angles as shown in eqn. (3)

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \frac{1}{c\dot{\theta}_2} \begin{bmatrix} c\dot{\theta}_2 & s\dot{\theta}_1\dot{\theta}_2 & c\dot{\theta}_1s\dot{\theta}_2 \\ 0 & c\dot{\theta}_1c\dot{\theta}_2 & -s\dot{\theta}_1c\dot{\theta}_2 \\ 0 & s\dot{\theta}_1 & c\dot{\theta}_1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \quad (3)$$

This is the simplified version of Euler's equations can be used on spacecraft for small maneuvers with high accuracy. [4]

#### A. Quaternions

Quaternions are four components, which express rotation about an axis. The first three components give information about the rotation axis and the fourth component gives information about the rotation angle. [5]

Quaternions have the advantage that they do not experience gimbal lock, yet they are less intuitive than Euler angles. There are 4-component vector, which express rotation about an axis. The first 3-components depict the Rotation axis of a vector while the 4th component depicts the rotation angle [6], [7].

$q = q_0 + q_4$

$$\begin{pmatrix} q \\ q_4 \end{pmatrix} \quad (4)$$

The above shown equation is to determine the quaternion components where  $\phi$  is the angle of Euler component.

#### B. Quaternion vs Euler

A comparative study Euler method is efficient because of linearized Euler angles and models are controllable, but there are many drawbacks in Euler method, which can overcome by quaternion method of attitude determination. Some of the major drawbacks faced by Euler method are:

1. Designs based on the linearized models may not globally stabilize original non-linear motion of spacecraft.
2. Euler method depends on the rotational sequences, which takes times in processing.
3. There is always a singular point where the model is not applicable.

The Quaternion method can overcome such problems easily as there is fast processing of vector in this method, the only disadvantage is that linearized quaternion model is not controllable, thus most depends on Lyapunov functions.

The Euler components can converted in quaternion components using the following set of equations.

$$\begin{aligned} \text{Quaternion1} &= s\left(\frac{\alpha}{2}\right) * c(\theta_1) \\ \text{Quaternion2} &= s\left(\frac{\alpha}{2}\right) * c(\theta_2) \\ \text{Quaternion3} &= s\left(\frac{\alpha}{2}\right) * c(\theta_3) \end{aligned} \quad (5)$$

## II. DYNAMICS OF SATELLITE

The coordinate system for locating satellite in time and space relative to earth and sun. Following that first order Perturbations has performed [8].

$$\begin{aligned} \text{Torque1} &= I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 \\ \text{Torque2} &= I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 \\ \text{Torque3} &= I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 \end{aligned} \quad (6)$$

$$\begin{bmatrix} \text{quaternion1} \\ \text{quaternion2} \\ \text{quaternion3} \end{bmatrix} = \begin{bmatrix} 0 & \cdots & \omega_1 \\ \vdots & \ddots & \vdots \\ -\omega_1 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \text{quaternion1} \\ \text{quaternion2} \\ \text{quaternion3} \end{bmatrix} \quad (7)$$

T is Torque

I is Moment of Inertia

$\omega$  is Angular velocity

$\Theta$  is Angular orientation

**Kinetic Energy:**  $E = \frac{1}{2} (I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2)$

**Magnitude of Angular Momentum:**  $M_2 = I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$

#### A. Perturbations

The orbit of a satellite has maintained accurately in for many active satellites. For example, a GEO satellite, which appear for fixed in the sky, must maintain its longitude and latitude position such that it does not leave its desired position. At the altitude of a GEO satellite, this corresponds to a square in space that is roughly 70 km in each side. Keeping a satellite in that square in the sky appears to be an easy task as this square is huge. However, in practice this is not true because of many factors [9], [10]. Some of these factors includes such as, the weightlessness of objects in space make them subject to the slightest forces acting on them. This means that when you are weightless, even a small force that acts on you in a specific direction will result in accelerating you in that direction. Over days, months, or even years, the accumulations of these accelerations become significant that a satellite may gain relatively high speeds in an undesired direction, and hence moving its location away from the target region.

The Keplerian model for the orbit of satellites as tried if Earth as assumed as a point mass at the center of its gravity. Clearly, this is not true because:

- The radius of Earth as you move over the equator changes [11].
- The density of Earth is not uniform but has higher values at specific regions causing a non-uniform gravitational force as a satellite moves in its orbit.

### B. Longitude Movement

The non-spherical shape and non-uniform density of Earth play a role on the motion of GEO satellites in the East-West direction changing their longitude position. Because of these, it has found that Earth has 4-equilibrium points: two stable equilibrium and non-stable equilibrium points [12]. A satellite that is placed at one of the stable equilibrium points will remain there while a satellite that is placed in one of the non-stable points will drift until it crosses the nearest stable point and remain oscillating around that stable point for a very long time.

### C. Latitude Movement:

Although the Sun is millions of times more massive than the Moon, the distance to the Moon is much smaller than the distance to the Sun, resulting in the gravitational pull by the Moon on a satellite being about 2 times the gravitational pull of the Sun. Since the pulls caused by gravitational forces of the Moon and Sun are not equal and they act at different directions as the satellite, the Sun, and the Moon all change relative positions, the resulting forces tend to move a GEO satellite in the latitude direction (North-South). The effect of these forces if uncorrected tend to move a satellite between Latitudes of 14.67° North and 14.67° South in a period of around 26.6 years. This movement changes the inclination of the satellite orbit by about 1° per year [12]

### D. Satellite Motion Effects:

An effect that is observed by result of the motion of satellites with respect to an observer that is usually observed with satellites in LEO or MEO satellites that is not observed with GEO satellites is the Doppler Effect. The motion that concerns us here is the one that causes the transmitter and receiver to either come closer to each other or move away from each other. Motion that does not cause the distance between the transmitter and receiver to change does not cause Doppler shift [13]. Since GEO satellites appear for fixed in space and the distance between them and a stationary Earth station does not change with time, no Doppler shift is felt with these satellites [14], [15].

The Doppler shift is given by (8)

$$\frac{f_r - f_T}{f_T} = \frac{\Delta f}{f_T} = \frac{v_{Relative}}{c} \quad (8)$$

- Design Specifications:

The performance specifications of various sensors and actuators were considered. By determining the yaw angle and providing necessary torque about the mentioned axis, the stability of control effectiveness of the system is determined. The same method can independently be applied to the other two

axes for achieving the desired control with reduced stability [16].

## III. METHODOLOGY

The code starts with the information about the spacecraft, which is simulated. This information consists of moment of inertia of the spacecraft, initial angular velocities, and initial Euler's angles. The periods of 90 minutes are simulated in which the Euler angles, angular velocities, quaternions and quaternion rates are calculated. In each iteration, the attitude matrix is calculated from both the Euler's angles and quaternions. The vectors, which are used for the Singular Value Decomposition, are of the horizon sensor and magnetometer. The magnetometer information is gained using the dipole approximation. Since any observation made will have an error, this error was simulated by adding noise to the reference vectors by the use of the approximate standard deviation of the sensor type, which is used.

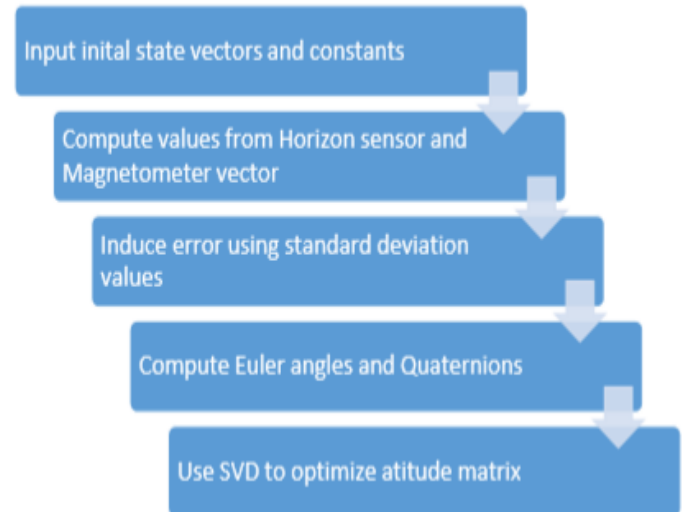


Figure. 3. Flowchart Representation of Attitude Determination Process

This inevitably changed and introduced error in the attitude matrix. To find an attitude matrix which is close to the original Singular Value Decomposition method has been used. The optimal, best approximation, attitude matrix has been obtained from Singular Value Decomposition. The optimal attitude matrix has been used to find the corresponding quaternions. These calculated quaternions are compared with the quaternions found by the use of the real values of Euler's angles.

- Wahba's Loss Function for SVD: Wahba's loss function is especially important when star trackers are used since star trackers track multiple stars simultaneously [17]

$$L(A) = 0.5 \sum_{i=1}^n a_i |b_i - A r_i|^2 \quad (9)$$

ai – weights  
bi – unit vectors (Body frame)  
ri – unit vectors (Reference frame)

SVD method has the added benefit of giving the eigenvector data as well as the eigenvalue data.

#### A. Satellite Attitude Control Algorithm

The complete mathematical model of satellite ADCS, a model of the individual control elements consisting of the Amplifier, DC motor and the satellite system were developed.

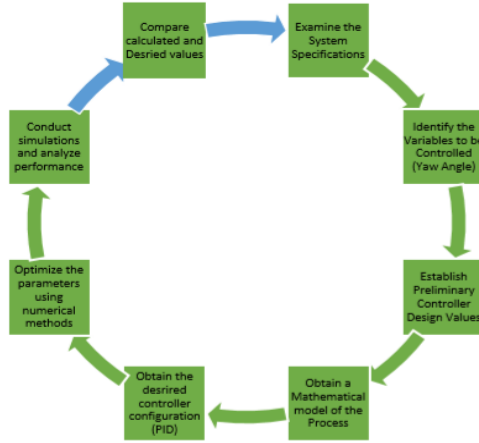


Figure. 4. Algorithm for Design of Attitude Stabilization System

#### Amplifier and DC Motor [17]

The open loop transfer function of Amplifier as given by –  

$$Vo/Vi = Ka \quad (10)$$

Where,  $Vo$  – Output Voltage

$Vi$  – Input Voltage

$Ka$  – Amplifier gain

The DC motor is the power actuator device that delivers output torque,  $T_m(s)$  from the motor. The input is the amplifier voltage output;  $Vo(s)$  is which supplies current,  $I_a$  to the resistance,  $R_a$  and inductance,  $L_a$  of the armature windings [18]. The input voltage has modelled in terms of the field or armature terminals. Here, we make use of the armature – controlled DC motor, which uses  $I_a$  as the control current.

#### The transfer function of the D.C. motor

$$Vo(s) = K (Js+b)(Ra+La(s))+K2 \quad (11)$$

Armature, inertia, electrical and electronic components, and amplifier gain. Thus, the following values were assumed –

TABLE I: Design data calculations

Specification	Values
Amp constant	10
DC Motor constant	0.01 Nm/Amp
Armature Resistance	1 Ohms
Armature Inductance	0.5H
MOI	0.01 Kgm2
Motor damping ratio	0.1
Satellite MOI	2.5kgm2
Satellite damping ratio	1.17

For the design of the controller and the optimization of its control parameters, it is imperative to select the values of satellite body inertia, the D.C motor gains [18]

#### IV. RESULTS

The optimization results has obtained for both attitude stabilization as well as attitude control using PID from attitude determination algorithms. The values of quaternions and an angular rate of the satellite has obtained initially by standard functions are compared and obtained from Singular Value Decomposition (SVD). This helps in eliminating the errors observed from set of values. The figure (5) & (6) shows the strong correlation between the given and calculated values, which indicates the accuracy of SVD over other methods.

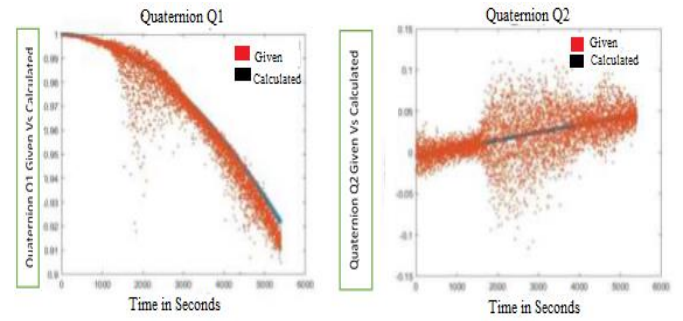


Figure. 5. Quaternion Q1 and Q2 Vs Calculated

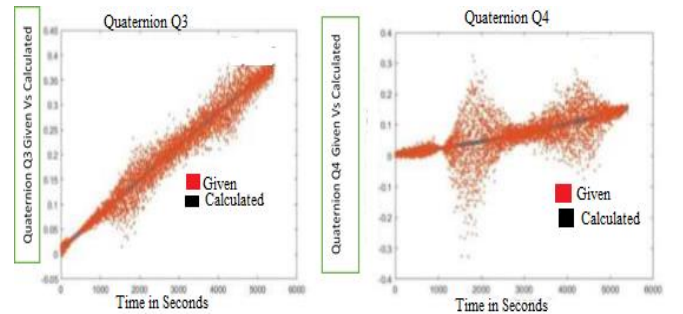


Figure. 6. Quaternion Q3 and Q4 Vs Calculated



From Figures (5) & (6) it has observed that the optimized Quaternions (Q1,Q2,Q3,Q4) values calculated by SVD follow the given values the initial period of 1000 seconds, deviate between period of 2000-4000 seconds, then show close correlation afterwards. The average error margin has found to be within 10.8 %, due to the high number of values iterated.

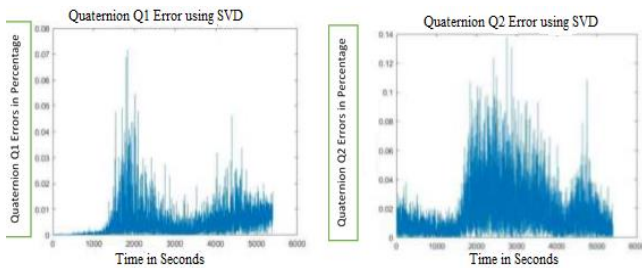


Figure 7. Quaternion Q1 and Q2 Errors

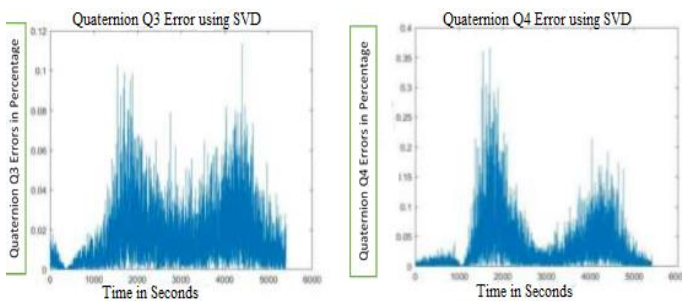


Figure 8. Quaternion Q3 and Q4 Errors

From Figures (7) & (8) it has observed that the Quaternions (Q1,Q2,Q3,Q4) errors.

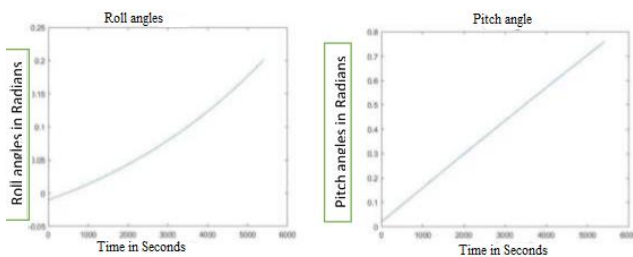


Figure 9. Roll and Pitch angle vs time

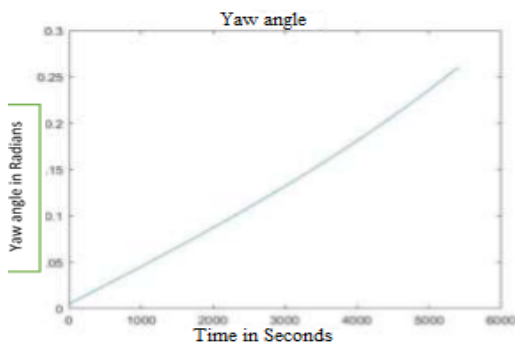


Figure 10. Yaw angle vs time

The pitch, roll and yaw angular rates (Figures 9 & 10) show a linear progression with time, which adheres to the initial set of values taken from standard satellite operations. It further indicates the merit of attitude determination using Singular Value Decomposition when angular rates need to as obtained by iteration of several vectors inputs from multiple sensors.

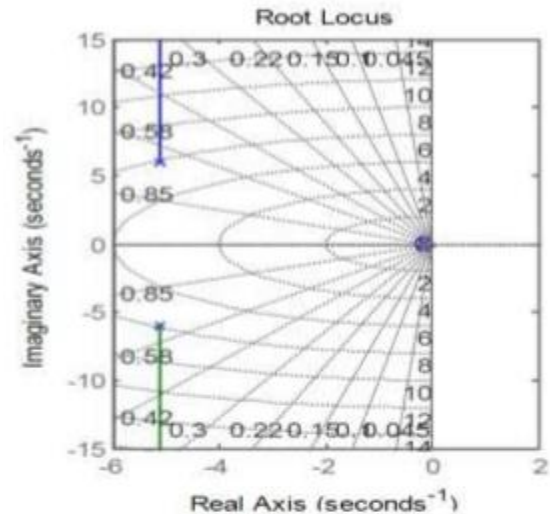


Figure 11. Root Locus plot for PID Control

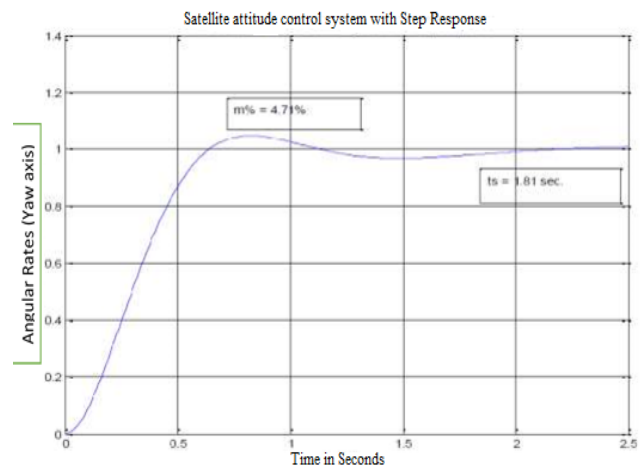


Figure 12. Plot depicting the rise time, overshoot and settling time

In Figure 11 shows the root locus analysis to calculate the controller gain for less percentage overshoot. The transient responses of system shown in Figure 12.

## V. CONCLUSION

In the simulation results, the satellite attitude control using quaternion (Q1, Q2, Q3, and Q4) rotations were calculated. The Singular Value Decomposition methods used to find the errors due to the perturbation forces in low earth orbits are discussed. The DC motor act as an actuator produces the control torque to satellite dynamics models. For the system,

one amperes currents generate the 0.01 Nm Torque to deflect the control surface of satellite. The transient response of satellite attitudes (Roll, Pitch, and Yaw) is calculated for ideal conditions. The step responses is maximum overshoot of 4.7 percentage and Settling time is 1.81 Seconds were achieved.

### ACKNOWLEDGMENT

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